Applied Algorithms, Summer 2017
Homework 2,
Due date: 12/7/17

1. (25 pts.) In class we saw an optimal algorithm for the Planar Single Facility Rectilinear Distance MiniSum problem (slides 7-9). For each of the following algorithms, either prove that it is another optimal algorithm for the problem or give an example to show it is sub-optimal.

Algorithm A: Let x*= weighted average of x-coordinate of the requests (= $\Sigma w_i a_i / \Sigma w_i$).

Let y*= weighted average of y-coordinate of the requests (=$\Sigma w_i b_i / \Sigma w_i$).

Returen (x*,y*)

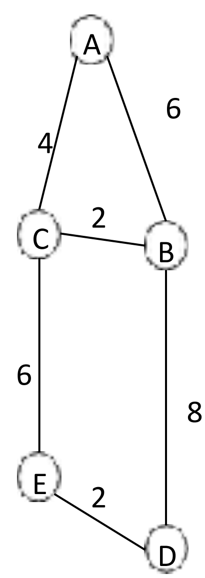
Algorithm B: Let x*= weighted median of x-coordinate of the requests (median of a sorted vector of length Σw_i in which a_i appears w_i times)

Let y*= weighted median of y-coordinate of the requests (median of a sorted vector of length Σw_i in which b_i appears w_i times)

If Σw_i is even, then select as median either the $\lfloor \Sigma w_i/2 \rfloor$ or the $\lceil \Sigma w_i/2 \rceil$ value.

Returen (x*,y*)

2. (25 pts.)
a. Solve the 1-center problem for the graph in the figure. Find both a single node center and an absolute center.
b. Solve the 1-median problem for the graph. Assume h(A)=3, h(B)=1, h(C)=2, h(D)= 2, h(E)=1. Remark: when finding the distance matrix, you don't need to actually run a shortest path algorithm, it is OK to conclude the values from the figure.



3. (25 pts.) Given is a star graph (a tree with depth 1) consisting of a middle node v and n>1 surrounding vertices. It is known that all vertices have positive weights and all edges have positive lengths. Prove or give a counter example for each of the following claims:

a. If all edges have the same length (and vertices have arbitrary weights) then v is the 1-median.

b. If all vertices have the same weights (and edges have arbitrary lengths) then v is the 1-median.

c. A new vertex is added to the graph and is connected by a new edge to one of the outer vertices of the star. If all vertices (including the new one) have weight 1, and all the star edges (but not necessarily the new edge) have the same length, then any 1-median of the graph is also a 1-center.

4. (25 pts.) As a result of the next high-tech crisis, your company is closed and you choose a new career in a farm. You have n potatoes with weights $b_1, b_2, \ldots, b_n$ grams that you wish to pack into packages. Each package must contain at least L grams of potatoes so you won't get sued for false advertising. Your goal is to maximize the number of packages that you fill to L or more grams. Consider the obvious greedy algorithm that considers the potatoes in an arbitrary order, and adds potatoes to the same package until that package contain at least L grams of potatoes. Assume that no potato weighs more than L.

a. Show that this is a 2-approximation algorithm. That is, the algorithm fills at least 1/2 as many packages as optimal.

b. Given m, describe an instance with $n=\Theta(m)$ potatoes for which the 2-ratio is as tight as possible.