Applied Algorithms, Summer 2017
Homework 1,
Due date:9/7/17

1. **(16 pts.)** Use the Traditional Marriage Algorithm to find a stable pairing of the following instance with n=4. Boys={A,B,C,D,E}, Girls={V,W,X,Y,Z} and preferences list as follows. Show your work in a table.

A: V,W,X,Y,Z	V: C,D,E,A,B
B: Z,Y,X,W,V	W: A,B,C,D,E
C: Z,Y,W,X,V	X: E,D,A,C,B
D: X,Z,Y,V,W	Y: B,D,E,C,A
E: X,Z,W,Y,V	Z: A,B,C,D,E

2. **(24 pts)** Consider an input to the stable pairing problem with n men and n women. Out of the n men, there are k smart men and n-k stupid men. Also, there are k smart women and n-k stupid women (for some k between 1 and n-1). Everyone would rather marry any smart person than any stupid person (so the first k entries in any preference list are of smart people of the opposite gender in some order). Prove that in every stable pairing, every smart man is matched with a smart woman.

3.a. (12 pts.) The problem $1||\Sigma jCj$ is solved optimally on an instance of n jobs. The solutions' value is z. The processing time of each of the n jobs is increased by 1. The problem $1||\Sigma jCj$ is then solved on the resulting instance. What is the solution's value, as a function of n and z. Explain.

3.b. (12 pts.) For some instance of n jobs, and m=2 machines, the value of an optimal solution to $P||C_{max}$ is z. The processing time of each of the n jobs is increased by 1. Prove or give a counter example: The value of an optimal solution for $P||C_{max}$ on the resulting instance is at least z + $\lfloor n/2 \rfloor$.

4.a. (16 pts.) Find an optimal solution for $P3||\Sigma_j C_j$ and the set of jobs of lengths {3,5,31,4,8,11,6,8,1,7}. What is C_{max} in your schedule?
What is the makespan of the schedule produced for this set of jobs and machines by list-scheduling (i) when the jobs are processed from left to right (ii) from right to left (ii) in LPT order?

4.b (20 pts.) Let n=zm (n= number of jobs, m = number of machines). Prove that in the optimal solution to $P||\Sigma_j C_j$ there are exactly z jobs on each machine.
Note: The algorithm SPT uses this property, so your proof cannot simply say ‘because this is what we get from SPT’).
Directions: Assume the claim is false. Therefore, there must be an optimal solution in which there is a machine M1 with z+x jobs (x≥1) and a machine M2 with z-y jobs (y≥1). Proceed using an exchange argument.