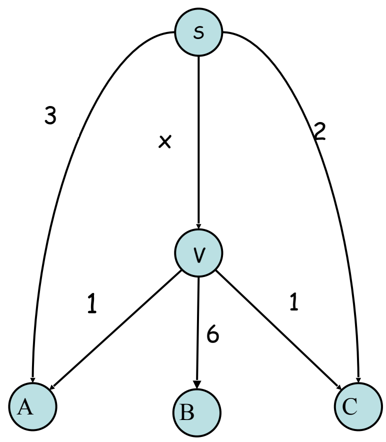
Applied Algorithms, Summer 2017
Homework 3,
Due date: 14/7/17

1. (35 pts) Three selfish agents A,B,C, need to send messages from s to the bottom nodes (one message to each target, the target of agent X is node X, for X∈{A,B,C}). What is the social optimum cost and what are the Nash equilibrium routings in the fair cost- sharing model?
Your answer should distinguish between different ranges of x (≥0). What is the price of stability and the price of anarchy for each range?
Note: There are 5 different ranges.



2. (35 pts.) On unrelated machines, the processing time of a job depends on the machine on which it is assigned. The input is given as an n×m matrix that includes the processing times p_{ij}. For a given assignment, the cost of a job is the total processing time of the jobs assigned to its machine.

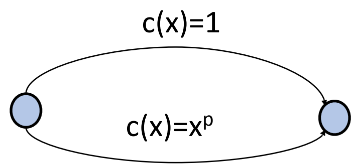
In this question you will prove that the *strong price of anarchy* for this model is at least m (the number of machines). The SPoA is the ratio between the makespan of the worst SE and the social optimum. Consider the following input (an example for m=5 is below): For each job J_j, j = 2, . . . ,m: $p_{j,j} = j$, $p_{j-1,j} = 1$, and $p_{i,j} = \infty$ for $i \neq j, j, -1$. For job J_1, $p_{1,1} = p_{m,1} = 1$ and $p_{i,1} = \infty$, for $i \neq 1, m$.

	1	2	3	4	5
1	1	∞	∞	∞	1
2	1	2	∞	∞	∞
3	∞	1	3	∞	∞
4	∞	∞	1	4	∞
5	∞	∞	∞	1	5

2.1. What is the makespan in the social optimum schedule (that achieves minimum makespan)?
2.2. Show that the schedule is which job j is assigned to machine j is a strong NE.
Hint: show by induction on j that job j is not part of a coalition.
2.3. Conclude that the SPoA in this game is at least m.
Remark: It is known that for any job scheduling game with m unrelated machines and n jobs, SPoA ≤ 2m−1.

3. (30 pts.) In class we saw that the Price of Anarchy for the following network with p=1 is 4/3.

Unfortunately, the ratio can be much worse when non-linear delay functions are allowed.
Assume that the delay on the lower edge is $c(x)= x^p$.
3.1. What is the flow at Nash equilibrium?
What is the total cost?
3.2. What is the optimal flow?
What is the total cost of the optimal flow?



3.3. What is the PoA as a function of p, and what is its value when $p \to \infty$?

Note: you will need simple calculus (derivative of a polynomial).